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A Genetic Algorithm Approach for a Real Life Fleet Size and Mix Vehicle Routing Problem: A Case Study in an Automotive Company

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Vehicle routing, fleet composition, mixed-integer programming, genetic algorithm.

Abstract
This paper considers the daily vehicle routing problem of an automotive company. The considered problem is a special case of the heterogeneous capacitated vehicle routing problem, the so-called fleet size and mix vehicle routing problem (FSMVRP), which decides on the type and number of vehicles required and minimizes the total cost comprised of hiring cost of vehicles, traveling cost and driver cost, under the service time constraint. First, the problem is formulated as a mixed-integer programming model and its validation is performed using small-sized problem instances. Next, a genetic algorithm (GA) based solution approach is designed and embedded into a user-friendly program developed for solving real problem instances in reasonable computation times. The computational results show that the proposed GA produces high-quality solutions. It also provides the decision maker the opportunity of evaluating alternative fleet compositions and distribution routes using the developed program.

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1. Introduction

The vehicle routing problems (VRPs) take place as the key decision area at the center of logistics management and they are modeled to find optimal vehicle routes under varying capacity, time, cost and/or distance constraints. The classical VRP consists of determining the best set of routes for a fleet of vehicles originating and terminating at a single central depot, to distribute goods to a set of customers, while minimizing the total distance/time or the total distribution cost.

In 1980s, the VRP was proved to be an NP-hard problem, that is, the solution time grows exponentially with the increase in distribution points, such that even the fastest computers are incapable of performing exhaustive computations required to determine an optimal solution (Wang and Lu, 2009). Therefore, many researchers have proposed and implemented metaheuristic algorithms, such as genetic algorithms (GAs) (Baker and Ayechew, 2003; Prins, 2004; Lacomme et al., 2006; Jeon et al., 2007), ant colony systems (Reimann et al., 2004) and particle

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swarm optimization (Ai and Kachitvichyanukul, 2009) in order to get a close to optimal solution for the VRP.

In this paper, the so-called fleet size and mix vehicle routing problem (FSMVRP) of an automotive company is solved. The FSMVRP is an extension of the capacitated VRP where the routes are restricted by the capacity of the vehicles and a heterogeneous fleet of vehicles is accommodated, and also vehicle activation costs are considered in addition to travel costs. The considered automotive company imports car parts and accessories from Europe to a central warehouse in Izmir, TURKEY and distributes them to customers in different cities on a daily routine, giving decisions on fleet composition and routing of vehicles considering problem specific operating constraints and cost components.

The main purpose of the study is to automate the solution of the FSMVRP of the company because it is very time consuming when handled manually as it is now. Also, actualized vehicle utilization rates are most of the time below the targeted level of the company. For this reason, a GA-based solution approach is designed to assist in the routing task and increase the vehicle utilization rate by identifying cost saving opportunities. Following, a computer program is developed with a graphical user interface that automates the routing task.

The remainder of this paper is organized as follows: Literature review about the FSMVRP is given in Section 2. Problem definition and mathematical model are presented in Section 3. The proposed GA approach is detailed in Section 4. Computational study and a real problem illustration are given in Section 5. Finally, concluding remarks are provided in Section 6.

2. Literature Review

The VRP embraces a whole class of complex problems, in which a set of minimum total cost routes must be determined for a number of resources (i.e. a fleet of vehicles) located at one or several points (e.g. warehouses) in order to service efficiently a number of demand and/or supply points. By this definition, one can make an inference that VRPs have many variants and modeling types, as well as they have wide application areas.

Ganesh et al. (2007) give a broad classification of VRPs by five views and mention the problem variants as well as solution methods. Accordingly, these views can be summarized as follows: i) Static conditions: single depot or multiple depot. ii) Vehicle related constraints: homogeneous capacitated, heterogeneous capacitated VRP or including fleet composition decision as mixed fleet. iii) Type of operations: pure pickup, pure delivery, pickup and delivery etc. iv) Problem features: deterministic or stochastic VRP whether demand and travel time data known or unknown in advance. v) Operational constraints: with regard to driver schedules, timing and load restrictions; these can be periodical VRP, VRP with time windows, VRP with backhauls.

There are many studies on the VRP and its above mentioned variants in literature. Since we deal with a real life FSMVRP, they are not exactly similar to our problem. However, herein we give a brief review of some of the studies which guided us during both modeling and solution stages.

Baker and Ayechew (2003) consider the application of GA to the basic VRP, in which customers of known demand are supplied from a single depot. Vehicles are subject to a weight limit, and in some cases there is a limit on the distance traveled. The computational results are given for both pure GA and a hybridized version of this GA with neighborhood search methods, showing that this approach is competitive with tabu search and simulated annealing in terms of solution time and quality. Prins (2004) has developed an effective and simple hybrid GA for the VRP without trip delimiters. The proposed GA is hybridized with a local search procedure, and it can be adjusted for both homogeneous and heterogeneous VRPs. Baldacci et al. (2008) give a survey of approaches from the literature to solve heterogeneous VRPs with fixed or variable fleet size, and focuses particularly on lower bounds and heuristic algorithms for these problem. Calvete et al. (2007) investigate the use of goal programming to model a general medium-sized delivery problem with soft and hard time windows where a heterogeneous fleet of vehicles and multiple objectives are considered.

The FSMVRP was first defined in a paper by Golden et al. (1984). Salhi and Rand (1993) review the early papers on fleet composition and emphasizes the shortage of literature that combines fleet composition with routing of the vehicles. Osman and Salhi (1996) give an overview of the papers regarding the FSMVRP up to that date. More recently, Hoff et al. (2010) describe industrial aspects of combined fleet composition and routing in maritime and road-based transportation, and presents the available literature. In literature, several exact solutions, constructive heuristics, and meta-heuristics exist. Yaman (2006) develops an exact solution for the FSMVRP when deriving formulations and valid inequalities to compute lower bounds to the problem. Six different formulations are developed, of which first four are based on Miller–Tucker–Zemlin constraints (1960) and the last two are based on flows. It is shown that the solutions obtained from these formulations are of good quality. Pessoa et al. (2007) present another exact algorithm that uses a branch-and-cut algorithm by introducing new powerful cuts. Baldacci et al. (2009) present a MIP formulation for the FSMVRP with fixed unit running costs. Some of the studies propose constructive heuristics to solve the FSMVRP (Golden et al., 1984; Gheysens et al., 1986; Salhi and Rand, 1993). There are also meta-heuristics proposed such as Gendreau et al. (1999) and Wassan and Osman (2002) which use tabu search and Liu et al. (2009) which propose a GA-based heuristic. As asserted in Hoff et al. (2010) a vehicle is closely related to a driver, or more generally, the crew. In the existing literature, the vehicle and driver form an inseparable unit: an equipage. It is possible that, in many applications, driving time restrictions will constrain driving. When this is the case, allocation and exchange of drivers may then be important aspects of the problem, which are included in this study.

The main features of our study that distinguish it from the previous studies can be described as follows: i) It considers a real life problem that involves both fleet construction and vehicle routes decisions with the objective of minimizing total operation cost. ii) The proposed MIP model takes all cost items into account, which are respectively vehicle activation cost, driver cost, and traveling cost of vehicles including their return to the central depot. iii) A GA-based solution approach, considering the specific operational constraints of the problem, is developed. iv)

The developed solution approach is embedded into a user-friendly program that allows the decision maker to evaluate many different input data simultaneously such as cost, customer service time, length of driver shift, and fleet composition.

2. Problem Definition

The automotive company gathers the car parts and accessories imported from Europe to a central warehouse in Izmir, which is the third biggest city of Turkey, to deliver to customers (dealers) dispersed in different cities in Turkey. The dealers send their orders via an online system to the warehouse and job orders are created for order picking. The orders are then packed and sorted by dealer cities to be loaded onto the vehicles for transportation.

Herein, the characteristics of the company's FSMVRP problem are given:

- The company does not have its own fleet and, therefore, it is a daily routine to decide on the number and type of vehicles to hire. The maximum number of vehicles from each type is determined according to daily freight and assuming that there is only one type of vehicle available for transportation.
- Each vehicle starts its route after being loaded at the main warehouse and returns to the warehouse after serving the customers.
- Customer demand, unloading times at customer sites and traveling times between each point in the transportation network are known.
- Each customer should be served by only one vehicle, and therefore each hired vehicle should have enough capacity for delivering at least one customer's demand.
- The total demand of customers on a particular route should not exceed the capacity of the vehicle assigned to that route.
- The maximum time each driver should deliver the goods to the customers on the assigned route should not exceed 72 hours.
- The total cost that consists of hiring costs of vehicles, driver costs pertaining to total working hours, and traveling costs of vehicles should be minimized.

3.1. Proposed Mathematical Model

A mixed-integer programming model (MIP) is developed for the company's FSMVRP. The notation used for the mathematical formulation is as follows:

Notation

n = number of customers; $i = 2,\ldots,n$, where ($i = 1$) represents the main warehouse

Q_i = demand of customer i

Ui = unloading time at customer i

F = maximum vehicle number, $k = 1,\ldots,F$

t_{ij} = traveling time from customer i to customer j

A_k = maximum capacity of vehicle k

c_k = hiring cost of vehicle k

c_{kij} = cost of traveling from customer i to customer j with vehicle k

$t_{\max}$ = maximum time that a driver should deliver the orders loaded on the vehicle (72 hrs in our case)

D_{ki} = total load of vehicle k on its route to customer i including the demand of customer i

t_k = total working time of the driver using vehicle k

s = shift duration in hours (8 hrs in our case)

w_k = driver cost per shift for using vehicle k

M = big number

Decision variables

x_{kij} = 1, if vehicle k travels directly from customer i to customer j, 0 otherwise

y_k = 1, if vehicle k is hired, 0 otherwise

The proposed MIP model is as follows:

$$\text{Minimize } \sum_{k=1}^{F} c_k\, y_k + \sum_{k=1}^{F} \sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=1}^{n} c_{ij}^{k}\, x_{ij}^{k} + \sum_{k=1}^{F} (w_k\, t_k)/s \qquad (1)$$

Subject to:

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=2}^{n} Q_j . x_{ij}^{k} \leq A_k y_k \qquad k = 1,\ldots,F \qquad (2)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=2}^{n} (U_j + t_{ij}) x_{ij}^{k} \leq t_{max}\, y_k \qquad k = 1,\ldots,F \qquad (3)$$

$$\sum_{j=2}^{n} x_{1j}^{k} = y_k \qquad k = 1,\ldots,F \qquad (4)$$

$$\sum_{i=2}^{n} x_{i1}^{k} = y_k \qquad k = 1,\ldots,F \qquad (5)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=2}^{n} (U_j + t_{ij}) x_{ij}^{k} + \sum_{j=2}^{n} t_{j1}\, x_{j1}^{k} = t_k \qquad k = 1,\ldots,F \qquad (6)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{k=1}^{F} x_{ij}^{k} = 1 \qquad j = 2,\ldots,n \qquad (7)$$

$$\sum_{\substack{i=1 \\ i\neq r}}^{n} x_{ir}^{k} - \sum_{\substack{j=1 \\ j\neq r}}^{n} x_{rj}^{k} = 0 \qquad r = 2,\ldots,n;\ k = 1,\ldots,F \qquad (8)$$

$$D_j^k \geq D_i^k + Q_j - M(1 - x_{ij}^{k}) \qquad i, j = 2,\ldots,n;\ i \neq j;\ k = 1,\ldots,F \qquad (9)$$

$$D_j^k \geq Q_j + \sum_{\substack{i=2 \\ i \neq j}}^{n} Q_i x_{ij}^k \qquad j = 2,\dots,n;\ k = 1,\dots,F \qquad (10)$$

$$y_k \in \{0,1\} \qquad k = 1,\dots,F \qquad (11)$$

$$x_{ij} \in \{0,1\} \qquad i, j = 1,\dots,n \qquad (12)$$

The objective function (1) minimizes the sum of hiring and traveling cost of vehicles and driver cost. Constraint (2) requires that the load of each vehicle that is compromised of the total demand of customers on the assigned route should not exceed the vehicle capacity. Constraint (3) ensures that the service time cannot be larger than the maximum time that the driver should transport the goods to the customers on the assigned route. Constraints (4) and (5) indicate that each vehicle, if hired, departs the central depot and turns back to the depot after completing its route. These two constraints can also be joined together by equalizing the left-hand sides for each vehicle k. Constraint (6) calculates the total working hour including the return time to the central depot for each vehicle's driver. Constraint (7) provides that each customer j should be served by only one vehicle. Constraint (8) is the formulation of typical flow conservation equation that ensures the continuity of each vehicle route. This equation implies that every demand point entered by a vehicle should be left by the same vehicle. Subtour elimination constraint is modeled in (9) according to Miller-Tucker-Zemlin (Miller et al., 1960; Kara et al., 2004) formulations. In relation to this, computation of each D_j^k is stated in (10) that replaces weaker requirements $D_j^k \geq Q_j$. The Big-M value has a smaller value than the loading capacity of the largest vehicle. For $x_{ij}^k = 1$ in constraint (9), then $D_j^k \geq D_i^k + Q_j$ as required. However, if $x_{ij}^k = 0$, then $D_j^k \geq D_i^k + Q_j - \mathrm{M}$, which is also satisfied since $D_j^k \geq Q_j$ and $D_i^k \leq \mathrm{M}$ (Yaman, 2006). The last two constraints (11)-(12) indicate the binary decision variables used in the model.

3.2. Computational Analysis of the Model

To evaluate the performance of the MIP model developed, several problem instances with varying number of customers are solved using LINGO software on a personal computer with Pentium(R) 4 CPU, 2.60 GHz, 1.00 GB RAM configuration. The transportation analyst in the company has noted that it takes about 3 to 4 hours to decide on the fleet requirements and prepare the daily route plans of vehicles. For this reason, the execution of the models has been limited to 4 hours for each problem instance.

Table 1 illustrates the computational results of the model. The gap values, that is, the relative percentage deviation of MIP solution values from the lower bound values obtained from the linear relaxation solution of the MIP model, are also given in the table. The optimum solutions have been achieved for problems with only up to 12 customers. As problem size increases, only feasible solutions could be found within the runtime limit, with no confirmation of optimality. For problems with more than 30 customers, the model could not even find a feasible solution. Considering the fact that the company may have to satisfy the delivery requirements of up to 39 customers daily, the proposed MIP model cannot fulfill the need to automate the daily vehicle routing activity.

For this reason, in the next section a GA-based solution approach has been developed for real size problems of the company in order to obtain optimum or near-optimum solutions in reasonable solution times.

Table 1. MIP model results obtained for the test problem instances

No. of Customers	Solution Time (min.)	MIP Solution Value		Lower Bound Value[a]	Gap (%)
8	0.42	4335.20	Optimum	4335.20	0.00
9	3.20	6394.10	Optimum	6394.10	0.00
10	5.90	4245.00	Optimum	4245.00	0.00
12	11.50	9382.75	Optimum	9382.75	0.00
14	240.00	7814.17	Feasible	7242.79	7.89
15	240.00	7524.00	Feasible	6686.00	12.53
16	240.00	9897.80	Feasible	8996.50	10.02
20	240.00	13138.60	Feasible	12840.90	2.32
25	240.00	18203.00	Feasible	15365.60	18.47
30	240.00	21459.90	Feasible	16152.70	32.86
35	240.00	-	-	20424.90	-
39	240.00	-	-	20794.60	-

[a] Lower bound values obtained from the LP relaxation of the MIP model

4. Genetic Algorithm Approach

GAs are population-based algorithms that simulate the evolutionary process of species that reproduce. GAs were first invented by Holland (1975) and developed by his colleagues and students at the University of Michigan. After then, Goldberg (1989) had provided some further developments. Mainly, GAs are search methods that perform the processes discovered in natural biological evolution. GAs cause the evolution of a population of individuals encoded as chromosomes by creating new generations of offspring through an iterative process that continues until some convergence criteria are met. At the end of this process, it is expected that an initial population of randomly generated chromosomes will improve and be replaced by better offspring. The best chromosome obtained by this process is then decoded to obtain the solution (Ganesh et al., 2007).

The population of individuals means a set of feasible solutions and each individual has a representation. The individual is also known as a chromosome that encodes a possible solution in a given problem or search space. The initial population is usually generated randomly in standard GAs. At every evolutionary step, called as generation usually, the individuals in the current population are compared and evaluated according to their fitness value for the objective function or target criteria of the problem. Crossover is the exchange of encoded solution parts (genoms or genes) between two selected individuals to produce offspring. Mutation means a permanent change (altering some genes) in chromosomes which leads to diversity in the population and helps preventing premature convergence; in other words, this variation avoids getting stuck at a local optimum value. Fitness value represents the quality level or proximity to target specifications of an individual. Thus, individuals or offspring with better fitness values are transformed into next generation.

The aim of GA is to reach to a high-quality solution for the given problem by using the genetic operators at every reproduction step. Although GAs do not always converge to the global optimum solution, this processing advantage lets GAs have efficient search techniques. Another main advantage of GA is its capability of manipulating encoded numerous chromosomes by parallel processing simultaneously. Since the whole solution space is searched at the same time, it seriously reduces the possibility of getting stuck in local optimum.

4.1. Structure of the Proposed GA Approach for the FSMVRP

A simple matrix representation for chromosomes is preferred since it does not require any specific encoding, and it is easier to read directly from the program developed. Basically, if there are n customers and m vehicles, a solution can be represented as a $2\text{x}(n-1)$ matrix since $n = 1$ shows the central depot. For example, with $n = 10$ cities and $m = 7$ vehicles consisting of different types, relevant solutions will be represented as follows:

Figure 1. Matrix representation for chromosomes

Parent 1

Customer	2	3	4	5	6	7	8	9	10
Vehicle	2	1	5	3	1	6	7	1	6

Parent 2

Customer	2	3	4	5	6	7	8	9	10
Vehicle	4	6	5	7	6	1	3	6	1

Regarding the construction of initial population, a large set of feasible 'L' solutions is obtained by a kind of 'sweep' approach that is similar to that of Baker and Ayechew (2003). Then, the best 'P' solutions are chosen to constitute an initial population of size 'P'. The steps of this approach are as follows:

- Customers are sorted in the ascending order of traveling costs to the central depot.
- A vehicle of any type is chosen randomly at the beginning and allocated to the first customer in line. The remaining customers are assigned to

the same vehicle in the sorted order as long as maximum capacity and service time constraints are not overridden.

- When a violation of any constraint occurs, the last customer assigned is moved to a new randomly chosen vehicle of any type, which is picked from the vehicle set.
- These steps are repeated until 'L' feasible solutions are obtained.
- Finally, the best 'P' out of 'L' solutions are chosen according to the fitness value for inclusion in the initial population.

For each chromosome in population, the objective function of the representative solution is calculated and recorded as the fitness value of the chromosome. Since the aim of the FSMVRP considered in this paper is to find the best routes with minimum total cost while service time and capacity constraints are satisfied, the chromosome with a smaller fitness value means a better solution for the FSMVRP problem.

An elitist tournament method is used for the selection of chromosomes that undergo crossover and mutation. According to Reeves and Rowe (2003), one way in which we could easily make GA converge is to keep a record of the best individual seen so far at each time step. Our selection procedure works accordingly as given below:

- Two pairs of chromosomes are chosen randomly from the initial population as reproductive parents.
- The fitness values of these pairs are compared and the first two chromosomes with the best fitness values are selected.
- The selected pair of chromosomes undergoes crossover to reproduce new offspring chromosomes.

A modified uniform crossover (UX) method is used to reproduce new solutions. As described by Golub and Picek (2010), UX generalizes definition of cross points to make every gene a potential crossover point. According to individual structure, it indicates randomly which parent will supply the offspring with which genes. Since UX performs random changes, it disrupts the individual genes with great probability but searches larger problem space. Hence, using this method is predicted to be effective while working on small sized populations. The pair of parents for crossover is selected by binary tournament method. Our crossover operator can be described as follows:

- Two customers (genes) are selected randomly on one parent and the serving vehicles of these customers are exchanged mutually with the serving vehicles of these customers on the other parent. In order to provide variation and obtain new offspring chromosomes, there must be different vehicles assigned to the customers in the selected genes of the other parent.

- The violation of capacity and service time constraints are immediately checked and, if they are all satisfied, the fitness values of the offspring chromosomes are compared with the fitness values of parents.
- The best two among these four chromosomes enter the population and replace the others, and so the population size remains unchanged.
- According to fitness evaluation, this replacement may include none of the parents if offspring chromosomes have better fitness values or none of the offspring chromosomes if they have worse fitness values than the parents, or if they do not satisfy the capacity and service time constraints.

An example for crossover operation between customers 3 and 8 is shown in Figure 2.

Figure 2. An example for crossover operation

Customers	2	3*	4	5	6	7	8*	9	10		2	3*	4	5	6	7	8*	9	10	
Parent 1	2	**1**	5	3	1	6	**7**	1	6	➔	2	6	5	3	1	6	3	1	6	Offspring 1
Parent 2	4	6	5	7	6	1	3	6	1		4	**1**	5	7	6	1	**7**	6	1	Offspring 2

Let us assume that the solution represented by Offspring 1 is not feasible since the capacity of vehicle 6 is exceeded by adding customer 3 on its route after crossover. Additionally, let the fitness values for the remaining solutions be in the order of Parent 2 > Parent 1 > Offspring 2. As a result, Offspring 2 enters the population by replacing Parent 2 and Parent 1 survives.

After crossover, survived parent(s) and accepted offspring(s) will undergo a simple mutation. The simple gene exchange procedure is proposed as mutation operator to gain variation among the remaining chromosomes. The mutation operator is applied as follows:

- Randomly select two chromosomes from the population and apply a simple mutation by selecting their two customers which are served by different vehicles and exchanging the vehicles mutually between these customers.
- If the obtained new chromosomes do not violate the capacity and service time constraints, their fitness values are calculated.
- The fitness values of mutated chromosomes are compared with those before mutation and the best two chromosomes are included in the population.

An example for mutation operation on customers 4 and 7 on Parent 1 and Offspring 2 is shown in Figure 3.

Figure 3. An example for mutation operation

Customers	2	3	4*	5	6	7*	8	9	10		2	3	4*	5	6	7*	8	9	10	
Parent 1	2	1	5	3	1	6	7	1	6	➔	2	1	6	3	1	5	3	1	6	P1m
Offspring 2	4	1	**5**	7	6	**1**	7	6	1		4	1	**1**	7	6	**5**	7	6	1	O2m

Let us assume that all of these chromosomes satisfy the constraints and the fitness values are ordered as P1m > Offspring 2 > Parent 1 > O2m. Thus, Parent 1 survives again and O2m participates in the population by replacing Offspring 2.

The procedure of the proposed GA is given in Figure 4 below:

Figure 4. The proposed procedure for GA

```
Create a set of feasible solutions by the proposed sweep algorithm
Select the best individuals from the feasible set to form the initial population
Repeat
        Select two pairs of chromosomes from the population randomly
        Determine two chromosomes with the best fitness values as parents
        Apply crossover to produce offspring solutions from the parents selected
        Check if capacity and service time constraints are satisfied by offspring solutions
            If satisfied, calculate the fitness values of offspring solutions and compare with parents
            Place the best two chromosomes into population and discard the other two
            End if
        Apply mutation operator to randomly chosen chromosomes from the new population
        Check if capacity and service time constraints are satisfied by the mutated chromosomes
            If satisfied, calculate the fitness values of new chromosomes
            Place the best two chromosomes into population and discard the other two
            End if
Until stopping criterion is satisfied
```

4.2. Programming of Genetic Algorithm in MATLAB

A program is developed in MATLAB language to implement the GA described in the previous section. This GA-based program has MS-Excel database connections for reading required input data from the addressed worksheets of an excel file, including customer demands, vehicle capacities, hiring costs, driver costs, traveling times and costs, and unloading times.

The company has 39 customers in Turkey but different numbers of customers may be served on each day according to the customer demand. Thus, our MATLAB program is designed to resize the traveling time and traveling cost matrices by eliminating the unnecessary columns and rows of the matrices belonging to those customers who have zero demand on a particular day. Before running the program, it is only expected from the analyst to input the daily customer demand and determine the maximum set of vehicles depending on demand data. In the vehicle set, the number of vehicles from each type is computed as shown below:

Number of Vehicle Type k = Total Daily Demand / Capacity of Vehicle Type k (13)

4.3. Parameter Settings

Test runs are made for the best parameter values regarding crossover rate, mutation rate, population size and required number of generations using 20-city and 35-city problem sizes. The same daily demand data and fleet composition is used for each group of test runs while population size and genetic operator rates are changed. In the light of past research regarding the values of genetic parameters, the crossover rate is chosen within the interval [0.25-0.70], and the mutation rate is chosen within the interval [0.05-0.50] with increments of 0.05. Population size varies from 25 to 40 with increments of 5.

The evolution graph of fitness function is plotted for each problem instance as illustrated in Figure 5 in order to observe the improvement against the generation number. Finally, the average improvement rates per parameter value of each problem instance are given in Table 2. According to the obtained results, for both problem sizes the same population size, crossover and mutation rates produce the highest average improvement rates.

Subsequently, the following parameter values are chosen: population size = 35, crossover and mutation rates are 0.60 and 0.40, respectively. Also, by the help of evolution graphs, it is observed that the developed GA does not require high generation numbers. From this point on, for problems with 20 customers and less, 15,000 generations will be used, whereas for problems with more than 20 customers, 30,000 generations will be preferred.

Figure 5. Evolution graph of a test instance in MATLAB program

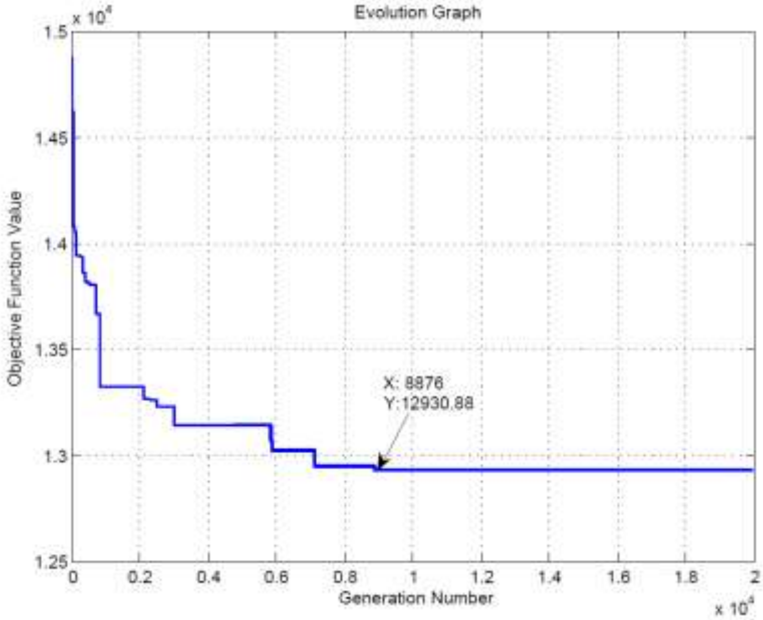

Table 2. Performance values for different parameter settings

	20 Cities	Improvement Rate (Average)	Max. Generation Number	35 Cities	Improvement Rate (Average)	Max. Generation Number
Population Size	25	13.68%	26,981	25	9.17%	25,396
	30	14.69%	18,295	30	10.42%	77,485
	35	15.24%	11,288	35	10.91%	25,192
	40	14.26%	15,868	40	10.24%	67,522
Crossover - Mutation Rate (%)	25 - 5	14.53%	11,137	25 – 5	9.04%	67,522
	30 - 10	14.14%	18,295	30 - 10	10.36%	36,994
	35 - 15	14.42%	26,981	35 - 15	9.99%	45,396
	40 - 20	14.08%	15,868	40 - 20	10.90%	41,547
	45 - 25	14.79%	8,714	45 - 25	10.96%	54,323
	50 - 30	15.41%	7,533	50 - 30	10.45%	14,866
	55 - 35	13.96%	4,417	55 - 35	9.60%	77,485
	60 - 40	15.84%	11,392	60 - 40	11.25%	29,403
	65 -45	12.93%	10,987	65 -45	9.54%	22,354
	70 - 50	14.57%	7,832	70 - 50	9.77%	10,648

5. Computational Study

Since the real life FSMVRP solved in this paper does not match any of the VRP studies in the literature, it is necessary for the company to design its own performance criteria. The transportation analyst has informed us that that the company aims for a vehicle utilization rate of 80% or above, which is calculated as the ratio of total demand over total vehicle capacity.

As it is shown in section 3.2, LINGO may not provide a global or even a local optimum solution in an acceptable runtime when the problem size gets bigger; however it gives a theoretical lower bound for the best objective function value. Table 3 illustrates the comparative results for the same problem instances given in Table 1, using both the MIP model and the proposed GA approach. The lower bound values noted from the MIP model for problems with no feasible solutions are used to give an idea about the quality of solutions reached by the proposed GA. It can be noticed that GA has achieved producing high-quality solutions in much reduced time compared to the MIP model, which is around 20 minutes for the largest problem instance.

The performance evaluation of GA has been further analyzed for problem groups with 15, 20, 25, 30, 35 and 39 cities. The actual data for five different working days has been extracted from the warehouse management system for these problem groups in order to solve five different problem instances belonging to the same group. By this way, the change in the performance of GA can be monitored for different problem sizes with different demand structures.

As can be seen in Table 4, vehicle utilization rates change slightly based on demand input. However, they are always above the targeted performance of the company, with a minimum vehicle utilization rate of 92.27%. According to the obtained results, it can be concluded that GA performs very well and provides high-quality solutions in reasonable computation times. As mentioned before, in the current system defining the fleet requirements and planning the vehicle routes take up to

3-4 hours every day for the analyst. On the other hand, GA finds solutions higher than the targeted vehicle utilization rates in rather short computation times.

Table 3. Performance comparison of GA with respect to MIP model

No. of Customers	Solution Value			Gap[a] (%) Solution Value	Solution Time (min.)		Gap (%) Solution Time
	GA	MIP			GA	MIP	
8	4335.20	4335.20	(Optimum)	0.00	0.25	0.42	-40.48
9	6394.10	6394.10	Optimum	0.00	0.70	3.20	-78.13
10	4260.40	4245.00	Optimum	0.36	4.17	5.90	-29.32
12	9695.10	9382.75	Optimum	3.33	4.25	11.50	-63.04
14	8214.70	7814.17	Feasible	5.13	5.10	240.00	-97.78
15	7544.00	7524.00	Feasible	0.27	6.30	240.00	-97.38
16	9815.80	9897.80	Feasible	-0.83	7.84	240.00	-96.73
20	12975.23	13138.60	Feasible	-1.24	8.37	240.00	-96.51
25	15977.50	18203.00	Feasible	-12.23	11.80	240.00	-95.08
30	17810.60	21459.90	Feasible	-17.01	14.23	240.00	-94.07
35	22020.03	-	Unknown	7.81[b]	18.23	240.00	-92.40
39	21986.55	-	Unknown	5.73[b]	22.24	240.00	-90.73

[a] Gap: (GA-MIP)/MIP

[b] GA solution values are compared to lower bound values obtained from the LP relaxation of the MIP model

Table 4. Performance evaluation for the problem groups based on demand of five different days

No. of Cities	**Problem No.**	**Solution Value**	**Solution Time (min.)**	**Vehicle Utilization Rate (%)**
15	1	7,912.64	6.84	92.27
	2	8,233.35	7.05	94.00
	3	7,971.30	6.73	97.48
	4	8,375.38	7.20	94.70
	5	9,142.84	7.66	95.80
20	6	13,857.69	9.47	95.73
	7	15,164.03	8.80	92.52
	8	15,505.40	7.96	98.74
	9	16,746.30	7.84	96.98
	10	12,975.23	8.37	93.33
25	11	15,325.59	12.16	97.30
	12	19,247.48	11.73	95.68
	13	19,171.94	13.01	95.02
	14	20,019.66	12.70	92.71
	15	15,977.55	11.80	97.43
30	16	16,008.65	13.57	98.63
	17	17,810.60	14.23	94.21
	18	19,373.12	15.53	95.00
	19	20,050.73	14.18	96.45
	20	20,192.94	15.52	97.48
35	21	22,020.03	18.23	95.87
	22	22,283.38	17.80	96.59
	23	24,146.22	16.20	94.49
	24	24,873.64	18.78	93.88
	25	20,622.01	17.63	96.08

Table 4. (cont.)

No. of Cities	Problem No.	Solution Value	Solution Time (min.)	Vehicle Utilization Rate (%)
39	26	18,564.73	20.18	93.30
	27	18,429.46	19.16	92.66
	28	22,875.84	20.80	94.09
	29	19,444.18	21.92	96.26
	30	21,986.55	22.24	95.53

5.1. A Real Problem Illustration

In this section, a real problem instance is given with the required input data and the solution found by the GA approach for this instance. Table 5 shows the customers in different cities, their demands, and unloading times at customer sites. There are three types of vehicles, which are respectively truck, lorry, and van. The number of vehicles for each type is calculated using Equation (13). The capacity of truck, lorry, and van is 4000, 2500, and 1500 m^3, respectively, hereas the rental costs are 1000, 650, and 450 Turkish Liras (TL), respectively. Finally, their driver costs per shift are 60, 45, and 32 TL for truck, lorry, and van, respectively.

Table 5. Customer Demand and Unloading Times for a Real Problem Instance

City	Demand (m^3)	Unload Time (h)	City	Demand (m^3)	Unload Time (h)
İzmir-Int	235	0.20	Kocaeli	1016	0.85
Manisa	804	0.67	Adapazarı	1008	0.84
Aydın	717	0.60	Tekirdağ	632	0.53
Balıkesir	469	0.39	Düzce	425	0.35
Uşak	863	0.72	Konya	3367	2.81
Denizli	706	0.59	İstanbul	2858	2.38
Muğla	3510	2.93	Ankara	674	0.56
Bursa	302	0.25	Zonguldak	37	0.03
Çanakkale	175	0.15	Çorum	1004	0.84
Isparta	1384	1.15	Kayseri	3240	2.70
Eskişehir	2019	1.68	Mersin	1429	1.19
Antalya	664	0.55	Adana	3520	2.93

The best solution obtained by GA has an objective function value of 15,325.59 TL, and it requires hiring five trucks and three lorries. We also show the routes of these vehicles starting from the main warehouse located in Izmir and returning back to İzmir as below.

Truck1: İzmir → Uşak → Eskişehir → Kocaeli → İzmir

Truck2: İzmir → İzm-Int → Manisa → Denizli → Isparta → Antalya → İzmir

Truck3: İzmir → Çanakkale → Tekirdağ → Adana → İzmir

Truck5: İzmir → Bursa → Kayseri → İzmir

Truck6: Izmir → Balıkesir → İstanbul → İzmir

Lorry1: Izmir → Zonguldak → Çorum → Mersin → İzmir

Lorry2: Izmir → Adapazarı → Düzce → Ankara → İzmir

Lorry3: Izmir → Aydın → Muğla

6. Conclusion

The purpose of this paper is to tackle the daily vehicle requirements planning and routing problem of an automotive company and design an efficient solution algorithm for the decision maker. The confronted problem is a special case of the FSMVRP problem. As far as the past research concerned, there are some studies which consider fleet composition and vehicle route construction at the same time, but they do not exactly consider the specific requirements of our problem. It is also noticeable that case studies on the FSMVRP are indeed very rare.

Initially, the studied problem has been formulated as a MIP model. The MIP model takes all cost items into account, which are respectively driver cost, vehicle hiring cost, and traveling cost. Since the model cannot solve real size problems, a specific GA has been designed and programmed in MATLAB for obtaining daily vehicle routing plans that yield higher vehicle utilization rates than the targeted level of the company. The proposed algorithm performs well and produces high-quality solutions in reasonably short computation times. The developed program has a user-friendly graphical interface that allows the decision maker to evaluate the changes in vehicle requirements and routing plans under changing data such as customer service time, traveling and unloading times, and costs.

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